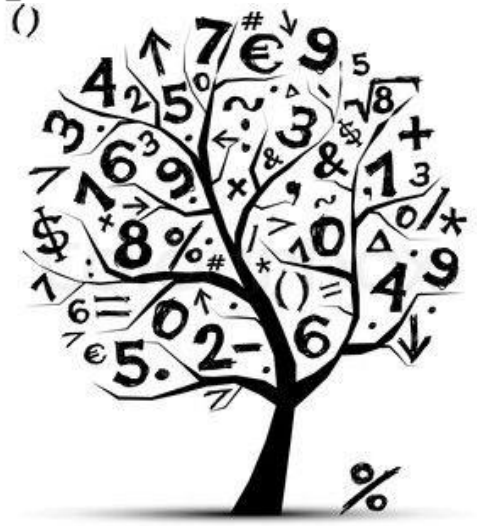
Calculations Policy

September 2019



Alvechurch C of E Middle School

Crown Meadow First School

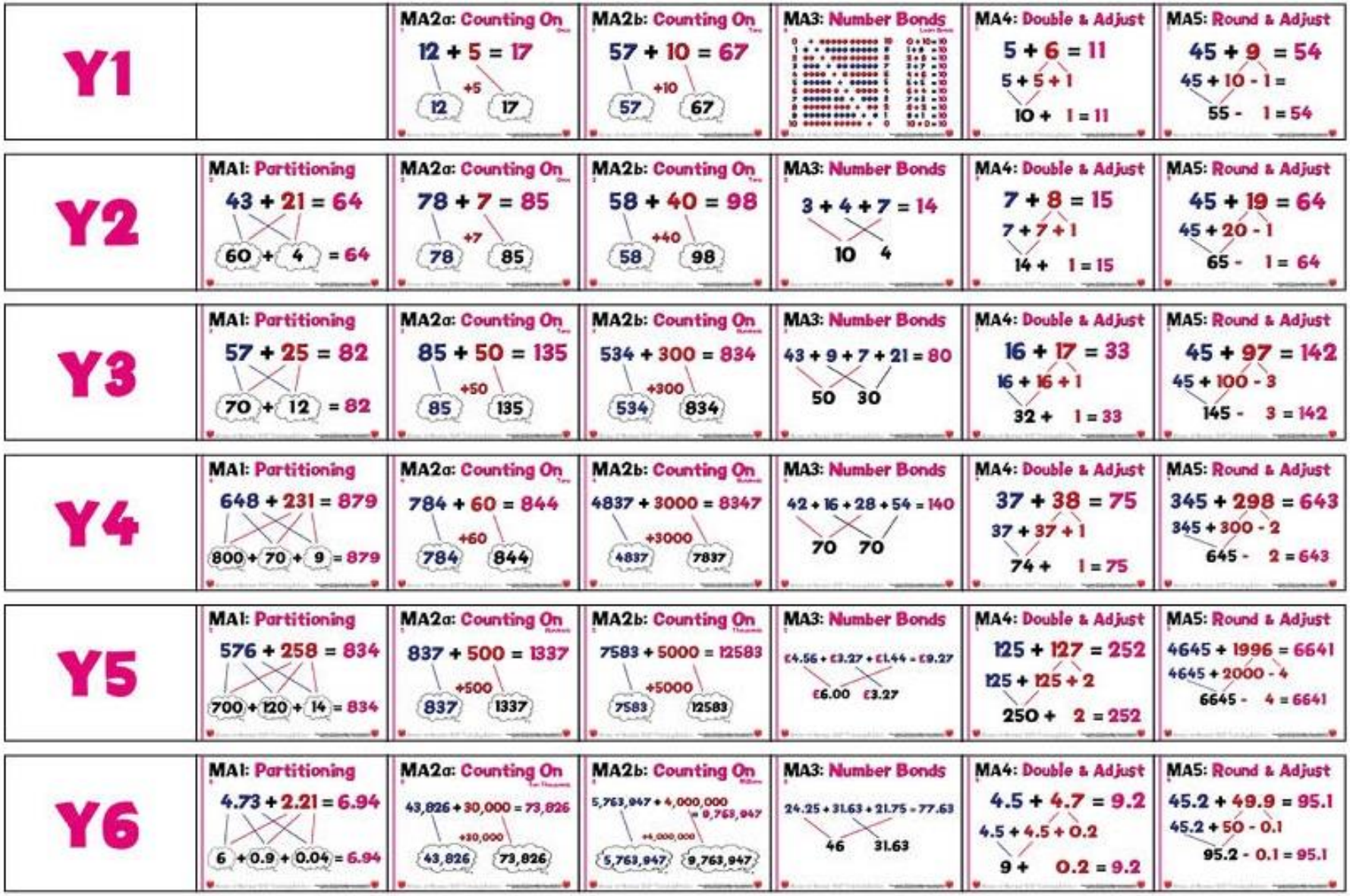
Blackwell First School

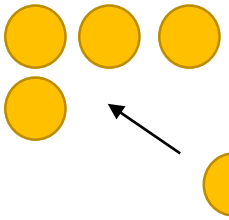
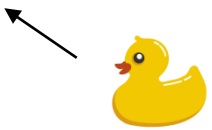
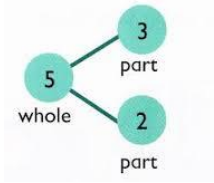
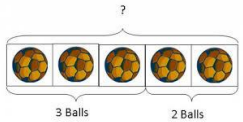
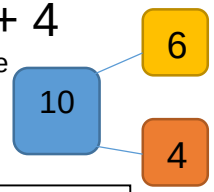
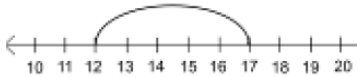
St Andrew's First School

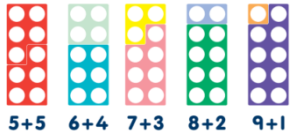
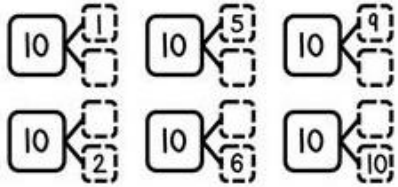
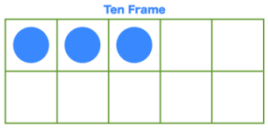
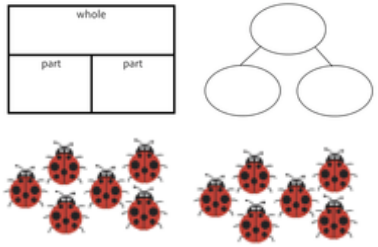
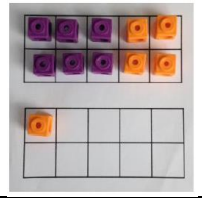
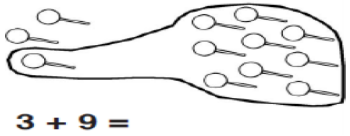
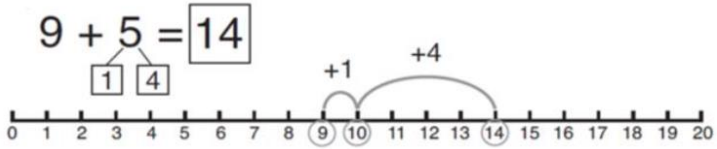
Addition

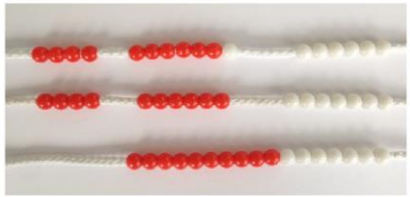
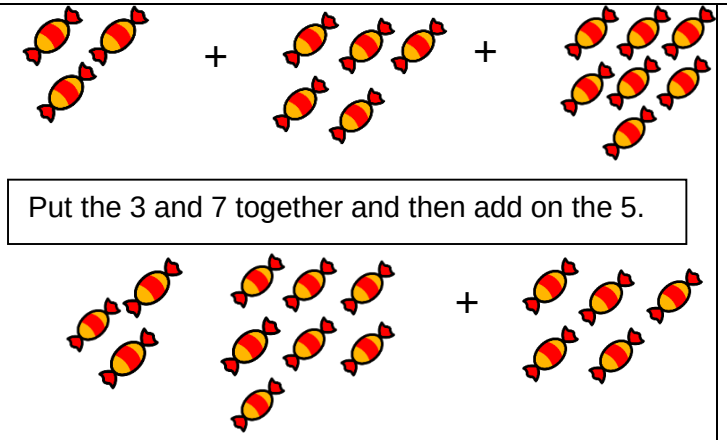
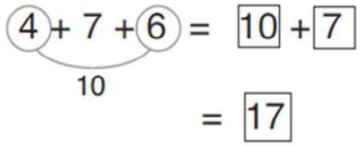
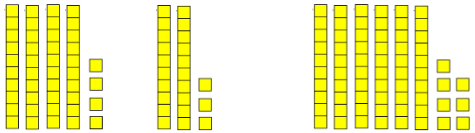
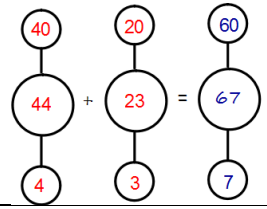
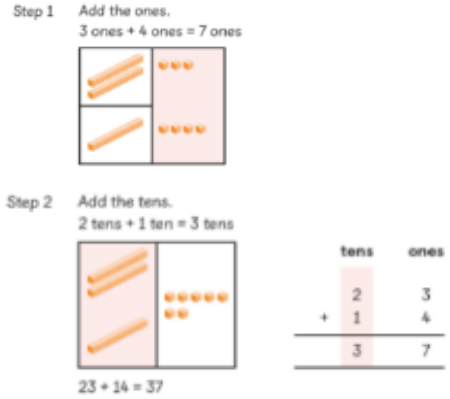
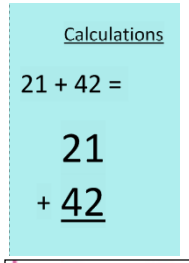
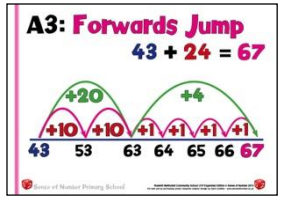
Mental Methods

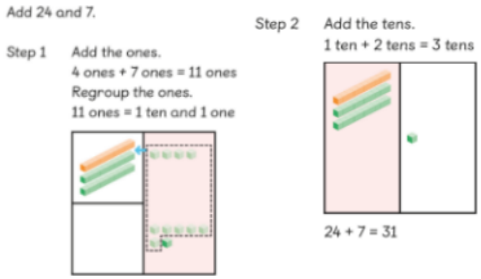
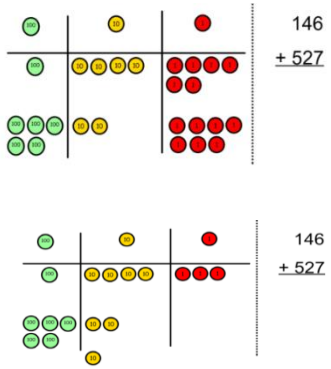
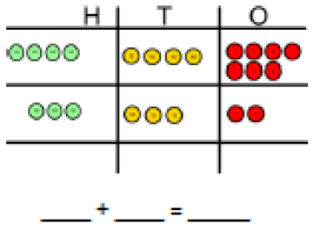
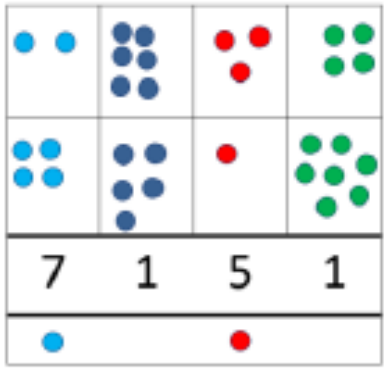
These mental arithmetic skills need to be taught discretely and then regularly practised in mental maths sessions to develop fluency. See attached ‘Sense of Number’ document.



Year group	Objective and Strategies	Concrete	Pictorial	Abstract
Reception	Adding one/ one more	Use concrete resources to show how 1 object can be added.  One more than 4 is 5	Draw more objects to show what has been added.  One more than 3 is 4 	
Reception and Y1	Combining two parts to make a whole	     Use cubes to add two numbers together as a group or in a bar. When using counters/cubes in a part whole model, combine the 2 amounts to put into the whole.	  Use pictures to add two numbers together as a group or in a bar.  Introduction of part-whole model from individuals squares/items to bars.  	Just Year 1 4 + 3 = 7 (say: 4 plus 3 is the same as 7) 10 = 6 + 4 (say: 10 is the same as 6 add 4)  Use the part-part whole diagram as shown above to move into the abstract.
Reception and Y1	Starting at the bigger number and counting on	 Reception use single digit numbers. 12 + 5 =? Start with the larger number on the bead string, abacus, egg box or tens frame and then count on the smaller number 1 by 1 to find the answer.	Year 1 only 12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	Year 1 only 5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

Reception and Y1	Knowing numberbonds to and within 10 from memory.	 Use numicon to show pairs of numbers to make 10. Year 1 begin to use this knowledge to make numberbonds to 20. Use base ten/dienes, egg boxes, tens frames and cuisenaire to show numberbonds to 10. Use unifix to make towers of 10:  Record number sentence as: 9+1 = 10 8+2 = 10 etc	 Use a part-part whole diagram to complete the missing number. Use ten frames: **3+__ = 10**, if you have 3, how many more to make 10? Children to draw tens frames. 	Reception – Verbal ____ goes with ___ to make 10 or ____ and ___ make ten. Year 1 0+__=10 1+__=10 2+__=10 3+__=10 4+__=10 5+__=10 6+__=10 7+__=10
Reception and Year 1	Recognise and understand doubles and halves. Year 1 recall doubles and halves to 10.	 Use a range of concrete materials to explore what doubling and halving is. Recognise examples and non-examples of doubles and halves.		Double 3 is 6 or 3 + 3 = 6 Half of 6 is 3 or half of 6 = 3
2	Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. Then add the rest to 10.	 Use pictures or a number line. Regroup or partition the smaller number to make 10. 	7 + 4 = 11 If I am at seven, how many more do I need to make 10? 7 + 3 = 10 How many more do I need to add on now? 10 + 1 = 11

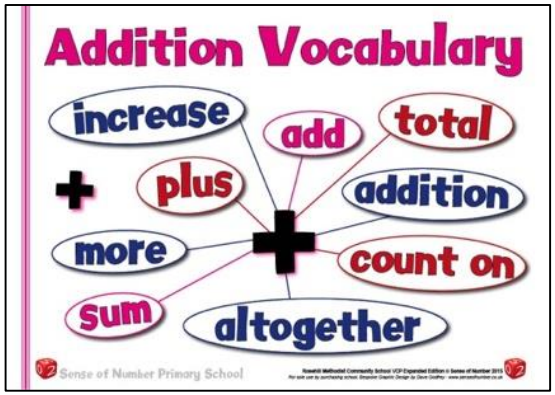
2	Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Then add on 7.  Or....spot doubles to add first, then add on the third number. Or....start with the largest number and add on the next largest number and then the third number.	 Put the 3 and 7 together and then add on the 5.	 Combine the two numbers that make 10 and then add on the remainder.
2	Adding two, 2-digit numbers by partitioning	44 + 23= Make the numbers using Dienes. Add together the tens and then add on the ones.  When manipulating numbers, group tens and ones into fives as a clearer visual representation for the children.	44 + 23= Draw the tens and ones. Count the tens and then add on the ones.	Partition each 2 digit number into tens and ones using the part-part whole model. Add together the tens, add together the ones and then recombine. 
2	Adding two, 2-digit numbers using the column method- no exchanging	24 + 15= Add together the ones first then add the tens. Use the base ten/Dienes.  	After practically using the base ten/Dienes children can draw the apparatus to help them to solve additions.	 Add the ones first and then the tens. 

Key Stage 2 Years 3,4,5, 6	Column addition method- with exchanging Y3 – up to 3 digit + 3 digit Y4 – up to 4 digit + 4 digit (formal written method) Y4 to be introduced to decimals in the context of money. Y5/6 – beyond 4 digit + 4 digit (formal written method) plus adding decimals	Make both numbers on a place value grid using Dienes or PV counters  Add up the ones and exchange 10 ones for a ten.  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Dienes to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can **draw** a pictoral representation of the columns and place value counters to further support their learning and understanding.   2634 + 4517 7151 This can also be done with Dienes.	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. Partitioned 200 + 30 + 6 + 400 + 20 + 3 600 + 50 + 9 = 659 57 + 25 12 70 82 + tens ones 2 7 1 5 4 2 1 536 + 85 621 1 1 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1

Vocabulary per year group:
Each year group should build on and consolidate previous year groups

ADDITION

Rec	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Part – a number of parts added together makes a whole **Whole** – a whole is made up of a number of parts **Equal** – say 'is equal to' or 'is the same as' **1 more than**	**Numeral** – how to write a number using digits **Digit** – 24 is a 2-digit number. The 2 represents the tens, the 4 represents the ones **Sum** – the total of one or more additions **Total** – the sum found by adding **10 more than.**	**Commutative** – addition is commutative so 8 + 2 = 2 + 8 **Inverse** – addition and subtraction are inverse operations so 7 + 3 = 10 and 10 – 3 = 7 **Exchange**– when adding ones if the total is greater than 10 we exchange 10 ones for a ten.	**Round and Adjust** – a mental strategy where one number is rounded to make the calculation easier and then adjusted e.g. 56 + 38 is treated as 56 + 40 and then 2 is subtracted to compensate **Exchange and carry** – when adding the ones in column addition if the total is greater than 10 we exchange 10 ones for a ten and carry to the tens column.	Consolidation of terms learnt in previous year groups	**Integer** – any of the positive or negative whole numbers **Positive** – any number greater than zero **Negative** – any number less than zero	Consolidation of terms learnt in all previous year groups



Mental Subtraction

MS1 – Manipulate the calculation (42 – 18 becomes 44 - 20)

MS2 – Round and adjust (52 – 19 becomes 52 – 20 + 1)

MS3 – Partitioning (45 – 12 becomes 45 – 10 – 2)

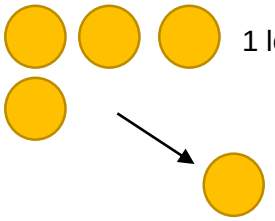
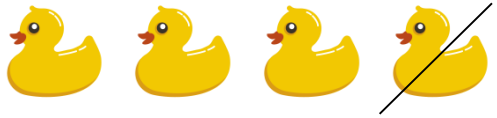
MS4 – Counting on (finding the difference)

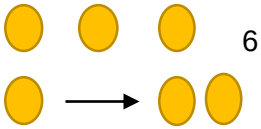
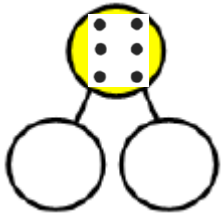
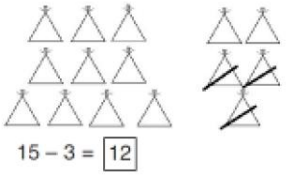
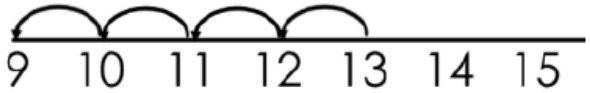
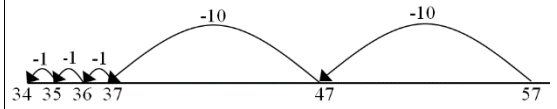
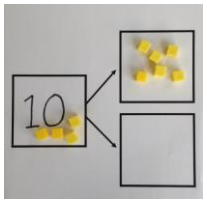
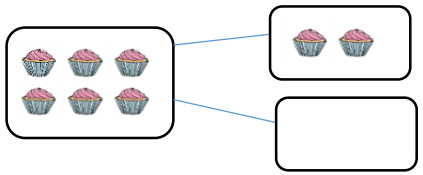
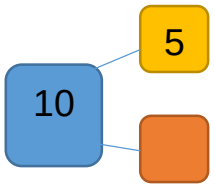
MS5 – Counting Back

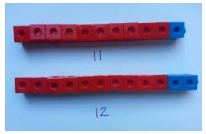
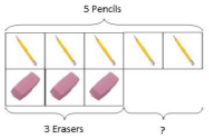
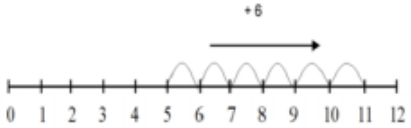
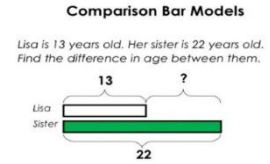
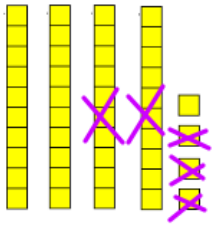
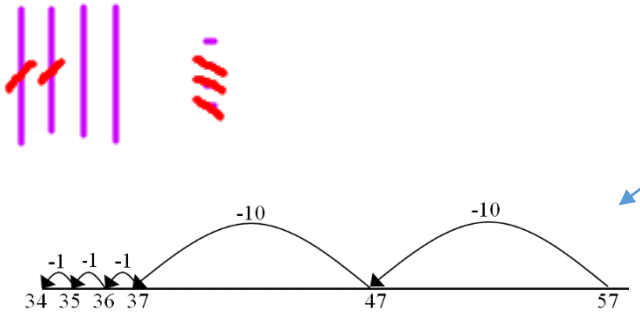
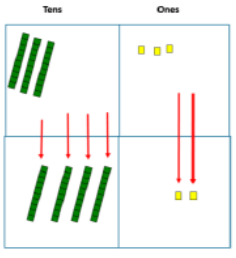
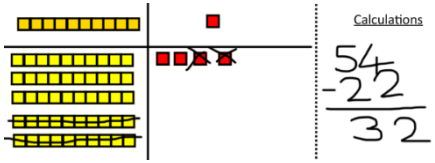
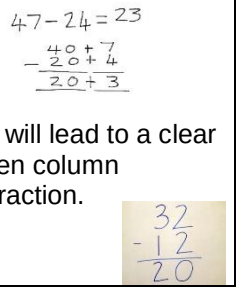
MS6 – Number facts (use what they know e.g. doubles, halves etc)

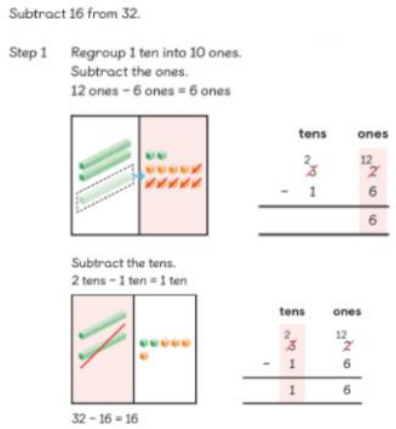
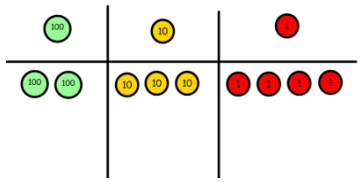
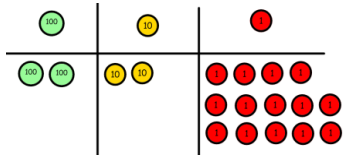
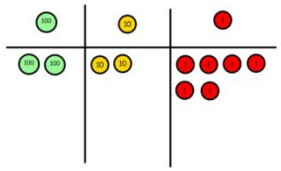
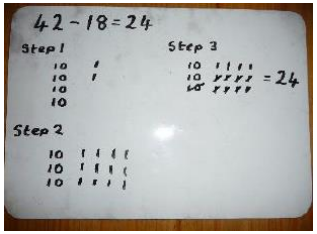
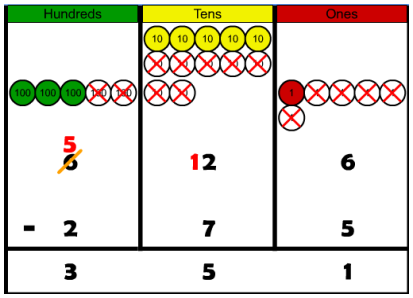
Subtraction

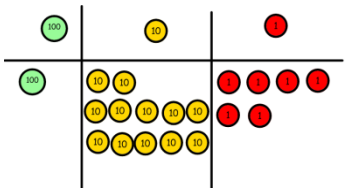
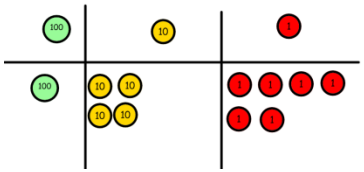
NOTE: When reading a subtraction question, use the vocabulary of calculation or equation and use 'subtract' when reading the symbol. Take away and find the difference are methods.

Year group	Objective and Strategies	Concrete	Pictorial	Abstract
Reception	Taking away one/one less	Use concrete resources to show how 1 object can be taken away.  1 less than 5 is equal to 4.	Cross out drawn objects to show what has been taken away.  1 less than 4 is equal to 3	

1	Subtracting ones	Use concrete resources to show how objects can be taken away.  6 – 2 = 4  6 – 2 = 4 When using counters/cubes in a part whole model, put in the whole number and then move the number to take away into one of the parts and the difference into the other part.	Cross out drawn objects to show what has been taken away.  15 – 3 = 12 14 – 5 = 9 	18 – 3 = ___ 8 – 2 = ___
1&2	Counting back	Make the larger number in your subtraction. Move the beads along your bead string or abacus as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track.  Start at the bigger number and count back the smaller number showing the jumps on the number line. 57 – 23 = ___  This can progress all the way to counting back using two 2-digit numbers. This can also be represented using drawings of base ten/Dienes with subtracted tens and ones crossed out.	Put 13 in your head, count back 4. What number are you on?
1& 2	Part-part Whole Model	 Link to addition- use the part-part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part?	Use a pictorial representation of objects to show the part-part whole model. 	

		10 - 6 = ___		Move to using numbers within the part whole model.
2	Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers. 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
2	Subtracting two, 2-digit numbers by partitioning	44 - 23= Make the first number using Dienes. subtract the second number. Count what is left. 	44 - 23= Draw the tens and ones. Cross out the second number. Count what is left. 	57 – 23 = ___ This can progress all the way to partitioning on a number line.
2/3	Column method -no exhanging	75 – 42 = ?  Use Dienes to make the bigger number then take the smaller number away. What's left? 32	 Draw the Dienes or place value counters alongside the written calculation to help show the working out.	 This will lead to a clear written column subtraction.

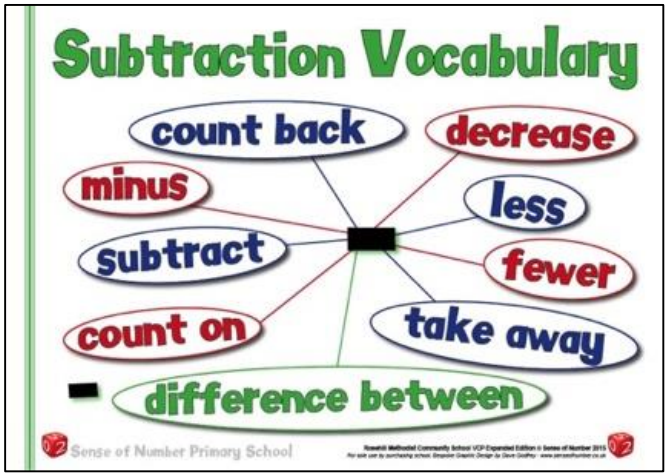
3,4,5, 6	Column subtraction method- with exchanging Y3 – up to 3 digit - 3 digit Y4 – up to 4 digit - 4 digit (formal written method) Decimal calculations to be introduced in the context of money. Y5/6 – beyond 4 digit - 4 digit (formal written method) plus subtracting decimals	Use base ten/Dienes to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.  Make the larger number with the place value counters  Calculations 234 - 88 Start with the ones, can I take away 8 from 4? I need to exchange a ten for 10 ones.  Calculations 234 - 88 Now I can subtract my ones.  Calculations 234 - 88 Now look at the tens, can I take away 8 tens? I need to exchange a hundred for 10 tens.	 When confident, children can find their own way to record the exchange. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.  Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. .	Children can start their formal written method by partitioning the number into clear place value columns. 200 + 60 + 3 -100 + 20 + 1 100 + 40 + 2 H T O 2 6 3 -1 2 1 1 4 2 Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals. 5 12 1 2 6 3 . 0 - 2 6 . 5 2 3 6 . 5

		 Calculations 234 - 88 Now I can take away eight tens and complete my subtraction  Calculations ~~2~~ ~~3~~ 4 - 88 146 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

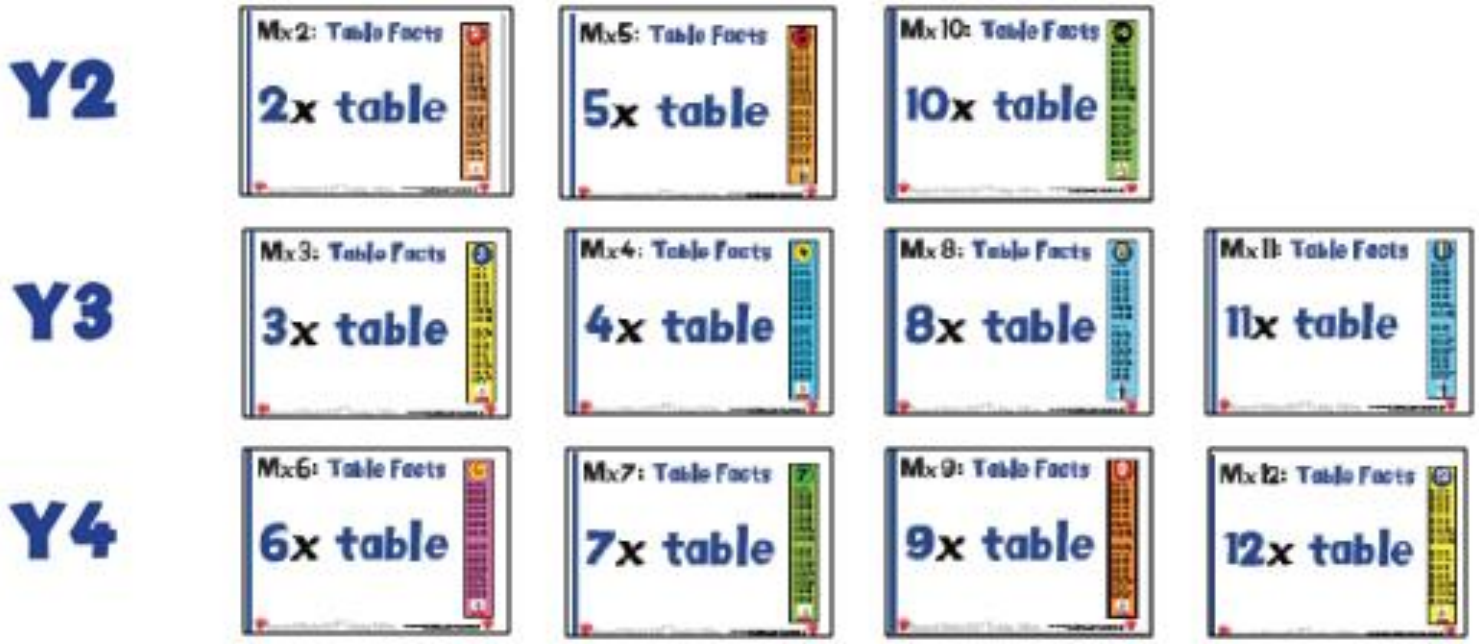
Vocabulary per year group:
Each year group should build on and consolidate previous year groups

SUBTRACTION

Rec	Year 1	Year 2	Year 3	Year 4, 5 & 6
Whole – a whole subtract any number of parts equals a part **Subtract** – to remove/**take away** a number of items from a group	**Subtract** – to carry out the process of subtraction **Minus** – a name for the symbol '-' **Difference –** the answer to a subtraction.	**Inverse** – addition and subtraction are inverse operations so 10 - 4 = 6 and 6 + 4 = 10 (it is NOT commutative) **Exchange** – when the number to subtract is smaller than the number we are subtracting from we exchange a ten into ten ones.	**Compensation** – a mental strategy where one number is rounded to make the calculation easier and then adjusted e.g. 56 - 38 is treated as 56 - 40 and then 2 is added to compensate	Consolidation of terms learnt in previous year groups



Multiplication – Mental Arithmetic



MM (Mental Multiplication)

MM1 – Manipulate calculation (Using arrays to identify component multiplications).

MM2 – Factorising (12 x 3 becomes 6 x 2 x 3)

MM3 – Reordering (numbers calculated in a different order – 9 x 2 x 5 is the same as 2 x 5 x 9)

MM4 – Partitioning (18 x 3 becomes 10 x 3 + 8 x 3)

MM5 – Round and adjust (49 x 3 becomes 50 x 3 – 1 x 3)

MM6 – Doubling (double 17 is equal to double ten + double 7)

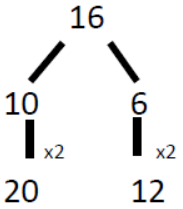
MM7 – Doubling table facts (16 x 7 becomes 8 x 2 x 7)

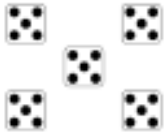
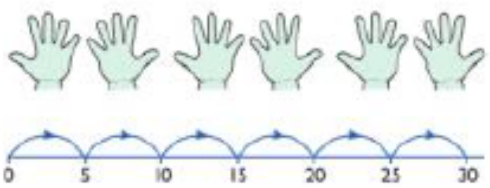
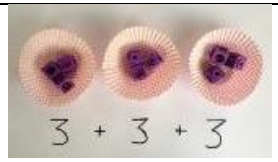
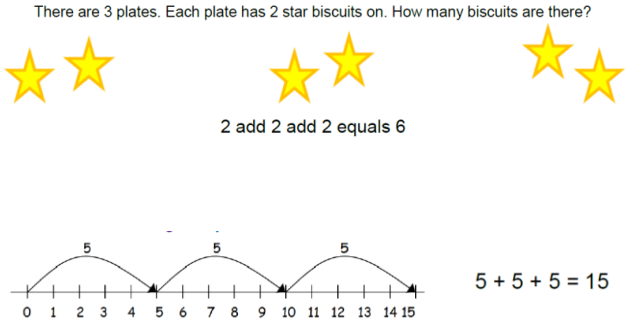
MM8 – Doubling up (36 x 4 becomes 36 x 2 x 2)

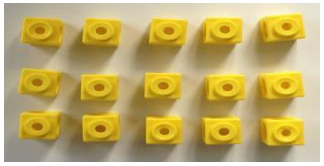
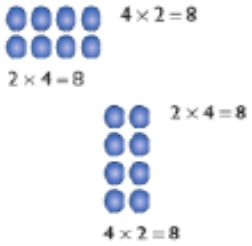
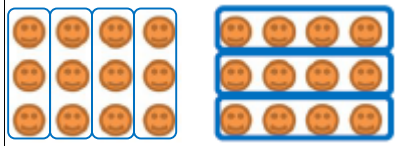
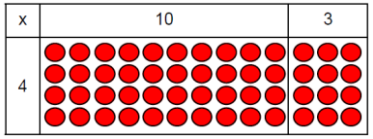
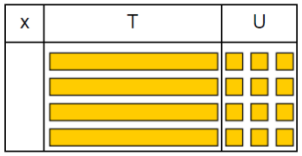
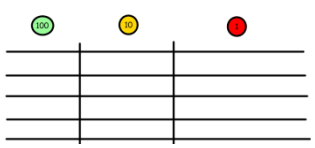
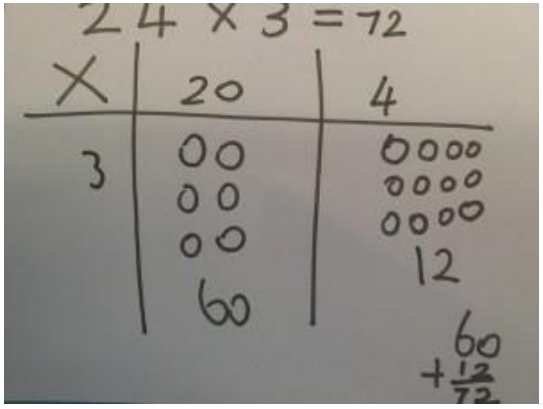
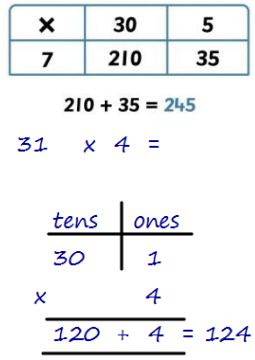
MM9 – Multiply by … then halve (86 x 5 becomes 86 x 10 then half)

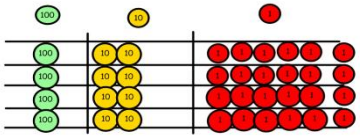
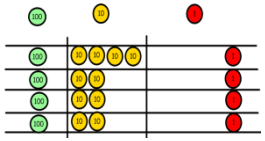
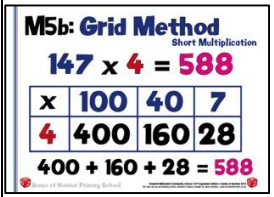
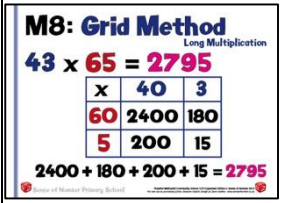
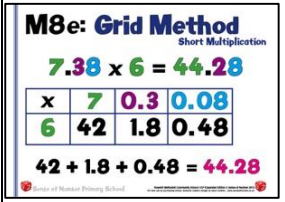
MM10 – Jump (multiply by 10, 100 and 1000)

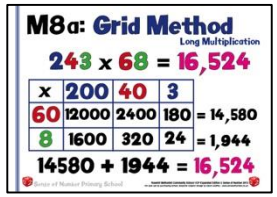
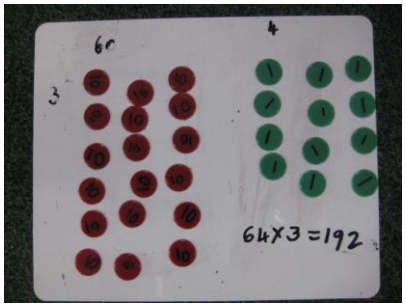
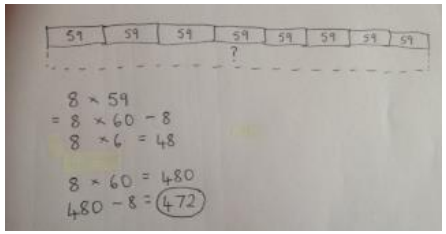
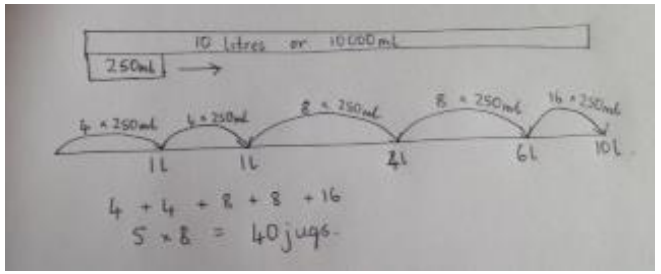
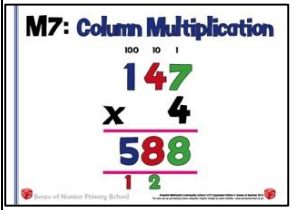
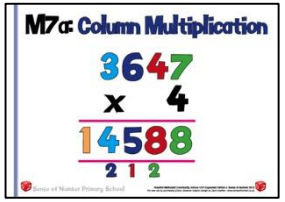
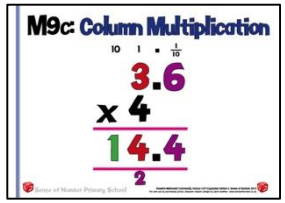
Multiplication

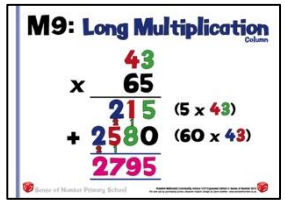
Year group	Objective and Strategies	Concrete	Pictorial	Abstract
1 and 2	Doubling	Use practical activities to show how to double a number.  Halves and doubles identified in a range of contexts, with a focus on equal halves. Shown on 10-frames and with Numicon. 	Draw pictures to show how to double a number. Double 4 is 8 	Year 1 – double 6 is 12 (doubles to 10) Year 2  Partition a number and then double each part before recombining it back together.

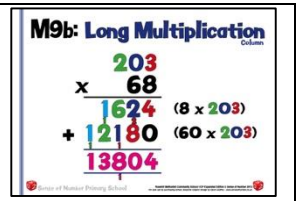
1&2	Counting in multiples/ grouping Y1 – count in 2s,5s and 10s Y2 – count in 2s, 3s, 5s from 0 and 10s from any number	   Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples. 12 3 3 3 3 4 groups of 3 = 12	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
2	Repeated addition	 3 + 3 + 3 =9  5+5+5=15 Use different concrete resources to add equal groups.	 There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there? 2 add 2 add 2 equals 6 5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10 2 x 5 = 10

2	Arrays- showing commutative multiplication Y2 – 2,5 and 10 times tables Y3 – 3,4 and 8 times tables Y4 – all tables up to 12x12	Create arrays using counters/ cubes to show multiplication sentences.  2 x 3 = 6  3 x 5 = ___	Draw arrays in different rotations to find **commutative** multiplication sentences.   3 x 4 = 12 and 4 x 3 = 12	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
3,4	Partitioning Y3 – 2 digit x 1 digit (mental and formal) Y4 – 2 digit x 1 digit 3 digit x 1 digit (formal)	Show the link with arrays to first introduce this method.  4 rows of 10 4 rows of 3 Move on to using Dienes to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.  Calculations 4 x 126 Fill each row with 126.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.  	Also :  Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

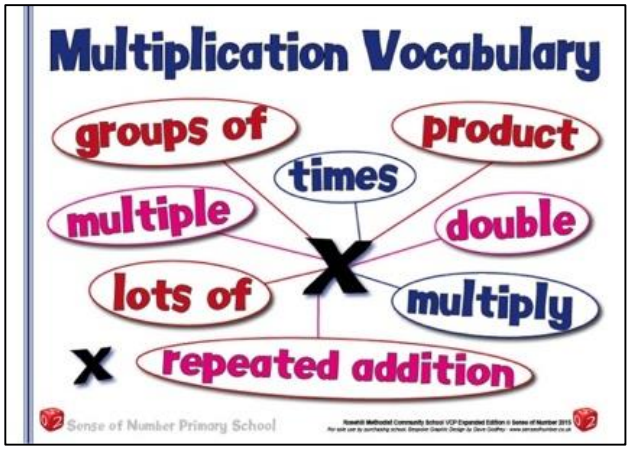
		 Calculations 4 x 126 Add up each column, starting with the ones making any exchanges.   Then you have your answer. 4 x 126 = 504		Y4 
5/6	Partitioning 4 digit x 1 digit (Those that can ove to 2 x 2 digit)			Y5   Y5/6 For 3 digit x 2 digit calculations, grid method is quite inefficient, and has much scope for error due to the number of 'part-products' that need to be added.

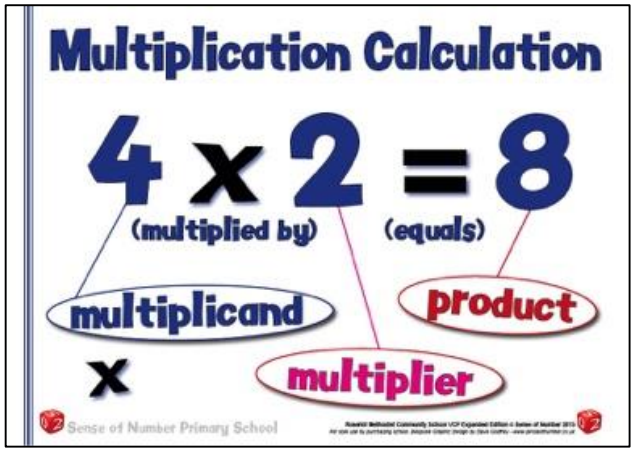
				Use this method when you find the standard method to be unreliable or difficult to remember. 
4,5,6	Column multiplication Y4 – 2 digit x 1 digit 3 digit x 1 digit (formal) Y5 – 4 digit x 1 digit 4 digit x 2 digit (formal) Y6 – 4 digit x 2 digit (formal) including decimals	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	Y4  Y5  Extend the use of standard method into the use of decimals. 

				With long multiplication, remind the children about lining up their numbers clearly in columns. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 If it helps, children can write out what they are solving next to their answer. This moves to the more compact method. Y5/6 Children should only use the 'standard' column method of long multiplication if they can regularly get the correct answer using this method. 

				 M9b: Long Multiplication Column 203 x 68 1624 (8 x 203) + 12180 (60 x 203) 13804

Vocabulary per year group: Each year group should build on and consolidate previous year groups					
MULTIPLICATION					
Rec	Year 1 **Groups of, sets of, lots of**	Year 2 **Multiply** – to carry out the process of multiplication **Multiple** – a number in a times table e.g. the multiples of 2 are 2,4,6 etc. **Groups of, lots of, sets of, times, multiplied by** – different ways to say the symbol "x" **Product –** the result of multiplying 2 numbers.	Year 3 **Factor –** factor x factor = product multiplicand x multiplier = product.	Year 4 **Factor –** Numbers we can multiply together to get another number. factor x factor = product e.g. 1,2,3,4,6,12 are factors of 12 **Factor pairs** - A factor pair is 2 factors multiplied together to make a given product	Year 5 & 6 **Prime number** – A whole number greater than 1 that only has two factors, itself and 1. **Composite** – a non prime number. **Common factor** – a number which is a factor of 2 or more other numbers e.g. 3 is a common factor of 9 and 30, 7 is a common factor of 14 and 21. **Prime factor** – the factors of a number that are prime e.g. 2 and 3 are the prime factors of 12 **Common multiple** – the smallest positive number that is a **multiple** of two or more numbers e.g. 24 is a common multiple of 4,6,8 etc.





Division - Mental Arithmetic

MD1 – Halving – partitioning.

MD2 – To divide by 4 or 8 use repeated halving.

MD3 - Manipulate the calculation. Adapt the calculation to make it easier. E.g. 140÷20 becomes 14÷2

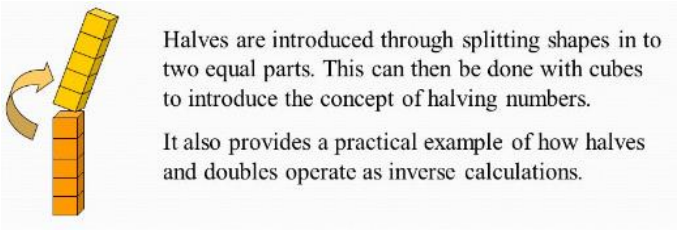
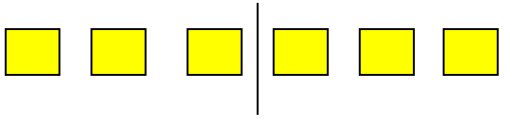
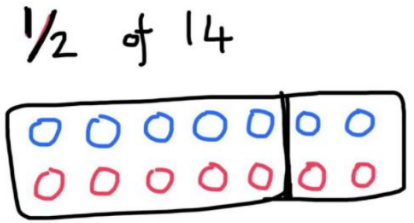
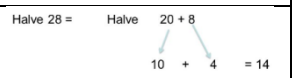
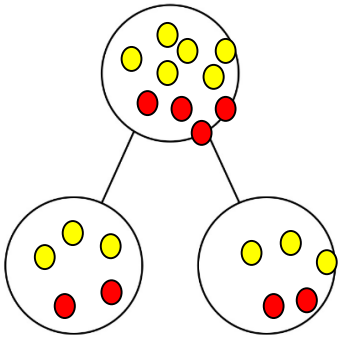
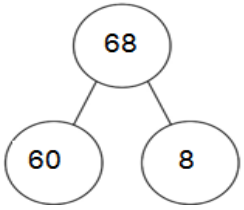
MD4 – To divide by 50, divide by 100 and then double. 3200 ÷ 50 becomes (3200÷100) x 2

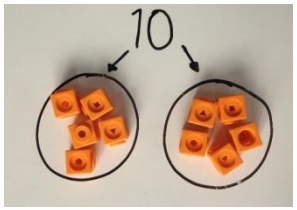
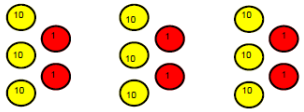
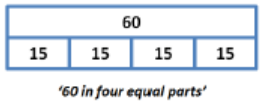
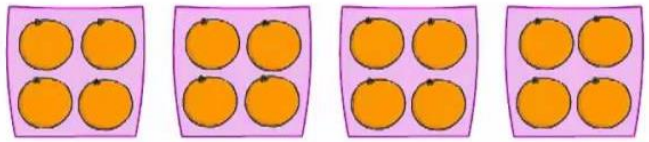
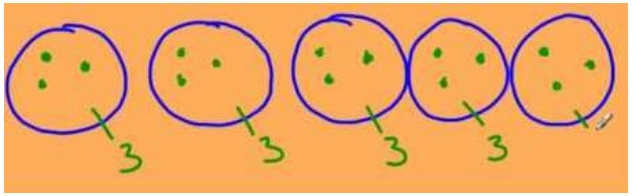
MD5 – To divide by 25, divide by 100 and then double twice.

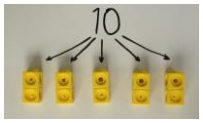
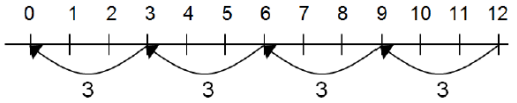
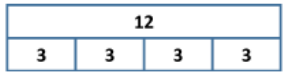
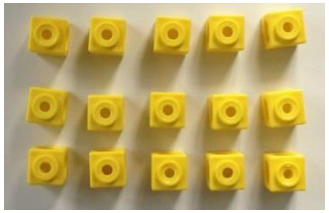
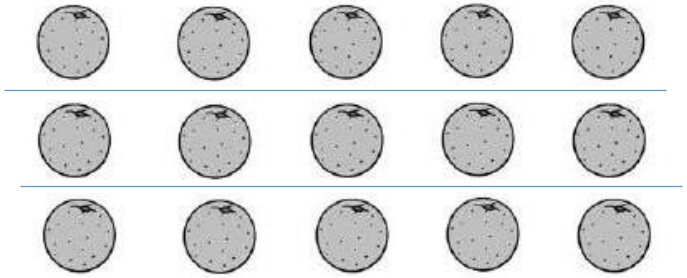
MD6 – To calculate fractions of numbers by dividing by the denominator and then multiplying by the numerator.

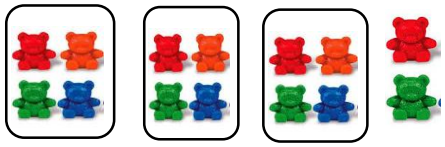
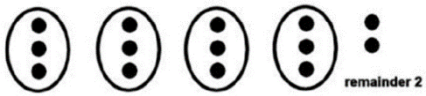
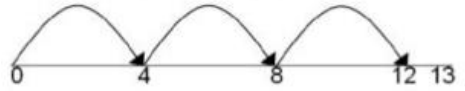
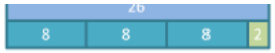
MD7 – To divide by 10, 100 or 1000 by moving digits to the right.

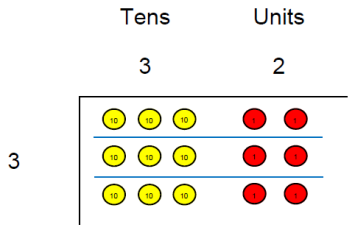
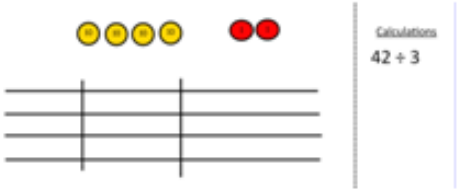
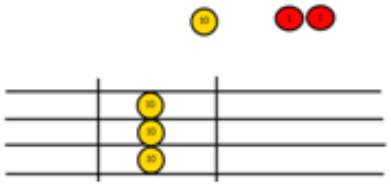
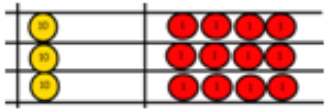
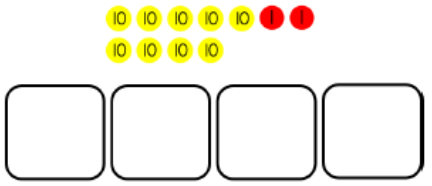
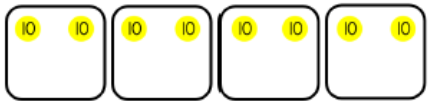
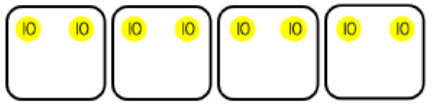
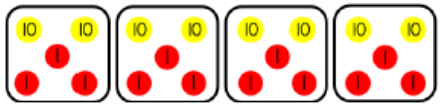
Division

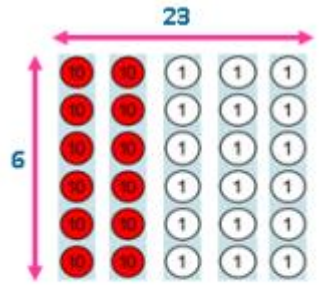
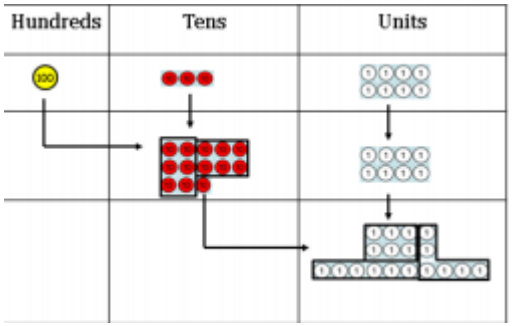
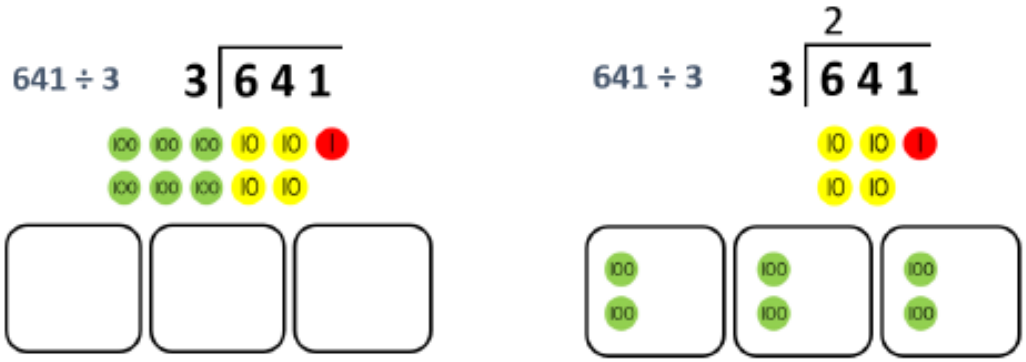
Year group	Objective and Strategies	Concrete	Pictorial	Abstract
R &1	Halving	Reception using concrete resources in a play context e.g. cakes onto plates.  Halves are introduced through splitting shapes in to two equal parts. This can then be done with cubes to introduce the concept of halving numbers. It also provides a practical example of how halves and doubles operate as inverse calculations.	Y1 Draw pictures to show how to halve a number.  Half of 6 is 3	
2	Halving	Use practical resources available. e.g. tens frames, egg boxes, dominoes, numicon		 Partition a number and then halve each part before recombining it back together.
3 4	Halving	 64 ÷2 = ? Partition the 2 digit number into tens and ones. Divide 60 by 2 and then 4 by 2. 60 ÷ 2 = 30, 4 ÷ 2 = 2 and 30 + 2 = 32	 68 ÷2 = ? Partition the 2 digit number into tens and ones. Divide 60 by 2 and then 8 by 2. Recombine. 60 ÷ 2 = 30 8 ÷ 2 = 4 and 30 + 4 = 34	56 ÷4 = ? Partition into 40 and 16 to divide each number by 4.

1,2	**Sharing** objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  (Y2) 8 ÷ 2 Think of the bar as a whole. Split it into the number of groups you are dividing into and work out how many would be within each group. Draw dots to represent initially. Bar model shows relationship between whole/ parts and makes links to division. 12 ÷ 4 = 3 	Share 9 buns between three people. 9 ÷ 3 = 3
3,4	**Sharing objects into groups.**	96 ÷ 3 = 32 	A pictorial representation of the place value counters or dienes.	60 ÷ 4 = 15  *'60 in four equal parts'* See short division(bus stop method as an abstract method for sharing).p28
R 1	Division as **grouping**	Divide quantities into equal groups. Use cubes, counters, objects. 	Y1 to draw 	

2,3	Division as **grouping**	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 10 shared into groups of 2 = 5 groups 10 ÷ 2 = 5 groups  12 ÷ 4 = 3  Grouping strategy modelled with covered arrays and Numicon: how many [divisors] in [dividend]? **20 ÷ 5 = 4** 20 dots. How many rows?	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  12 divided into groups of 3 = 4 groups. Leading into using an empty numberline. 12 ÷3 = 4 Bar model shows relationship between whole/ parts and makes links to division.  	30 ÷ 5 = ___ How many 5s make 30? 14 ÷ 2 = ___ How many 2s make 14?
2 3	Division as the inverse of multiplication	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 2 = 14 2 x 7 = 14 14 ÷ 7 = 2 14 ÷ 2 = 7

3/4	Division with a remainder	14 ÷ 3 = Divide objects between groups and see how many are left  So 14 ÷ 3 = 4 remainder 2 which can be written as 4r2. The concept of 'How many [divisors] in [dividend]' shown using Numicon, part-hidden arrays and by making shapes with matchsticks. **20 ÷ 3 (how many 3s in 20?) and 20 ÷ 5 (how many 5s in 20?):**  20 ÷ 3 = 6 r 2 whereas 20 ÷ 5 = 4	Draw dots and group them to divide an amount and clearly show a remainder.  So 14 ÷ 3 = 4r2. Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw a bar model: 26 ÷  8 = 3 r 2	Complete written divisions and show the remainder using r. 

3,4, 5,6	Short division Bus stop method can be division by sharing. Y3 – 2 digit ÷ 1 digit Y4 – 3 digit ÷ 1 digit Y5 – 4 digit ÷ 1 digit, including remainders as fractions or decimals according to the context Y6 – 4 digit ÷ 2 digit, including remainders as fractions or decimals according to the context	 Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	96÷ 3 = 32 Use place value counters to divide using the bus stop method.	92 ÷ 4 4 | 9 2  92 ÷ 4 2 4 | 9¹2  92 ÷ 4 2 4 | 9¹2  92 ÷ 4 2 3 4 | 9¹2 	Begin with divisions that divide equally with no remainder. 2 1 8 3 4 | 8 7 2 Move onto divisions with a remainder. 8 6 r 2 3 5 | 4 3 2 Finally move into decimal places to divide the total accurately. 1 4 . 6 16 21 3 5 | 5 1 1 . 0

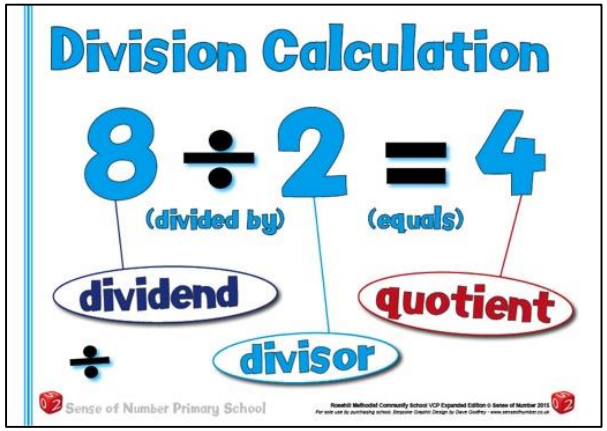
			138÷6  Check using multiplication inverse.	
5,6	Or place value counters using grouping.	138 ÷ 6 = 23  		$\begin{array}{r} 2\,3 \\ 6\overline{)\,1^{1}3^{1}8} \end{array}$
6	Formal written method of division – short division and long division Y6 – 4 digit ÷ 2 digit, including remainders as fractions or decimals	Division modelled with place value counters.  		

	according to the context		**SHORT DIVISION:** *No remainder:* 560 ÷ 4 = 140 *Remainder as a whole number:* 564 ÷ 5 = 112 r4 *Remainder as a fraction:* 112 4/5 *Remainder as a decimal:* 112.8 **LONG DIVISION:** *Remainder as a whole* 21 r10 *Remainder as a fraction in its lowest form:* 21 r 10/25 (2/5) *Remainder as a decimal:* 21.4

Vocabulary per year group:
Each year group should build on and consolidate previous year groups

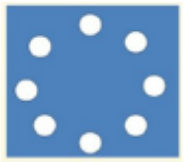
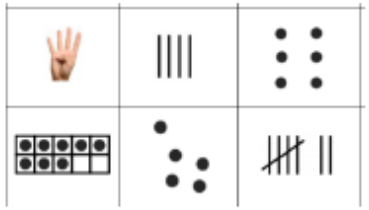
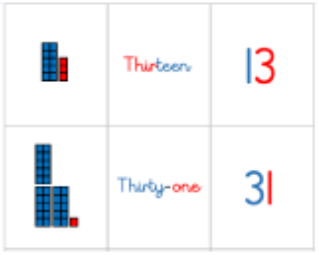
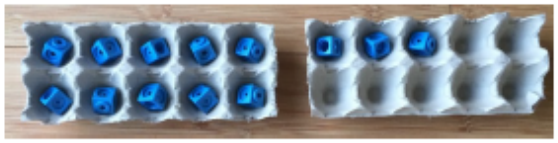
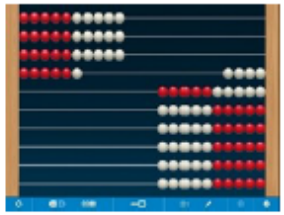
DIVISION

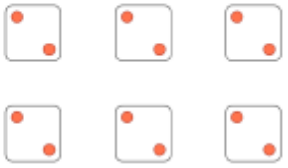
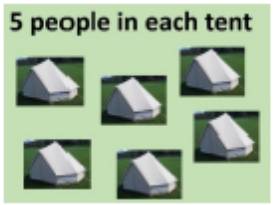
Rec & Year 1	Year 2	Year 3, 4, 5 and 6
Sharing – share equally a number of objects into a specified number of groups. **Divide** – to carry out the process of division	**Sharing** – sharing equally between **Grouping** - put into groups of **Divided by** – sharing or grouping **Inverse** – multiplication and division are inverse operations so 10 ÷ 2 = 5 and 5 x 2 =10 (it is NOT commutative)	**Dividend** – the number that is being divided into equal parts **Divisor** – for sharing: the number that it is being shared between. For grouping: the number in each group In 15 ÷ 3, 15 is the dividend and 3 is the divisor **Quotient** – the result of a division dividend ÷ divisor = quotient **Divisible** – A whole number is divisible by another if there is no remainder after division **Remainder** – the amount remaining after division e.g. 29 ÷ 7 = 4 r1



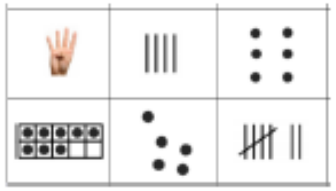
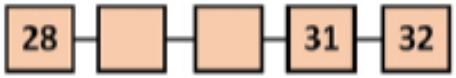
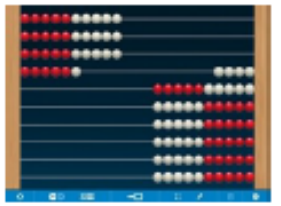
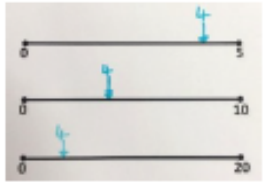
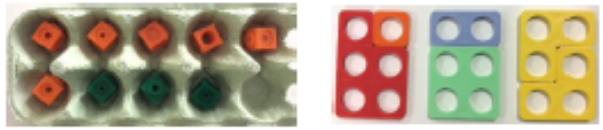
VISUAL REPRESENTATIONS

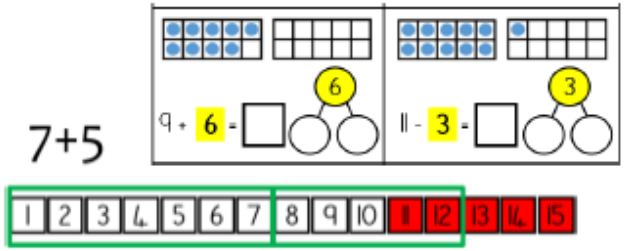
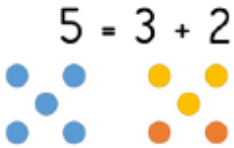
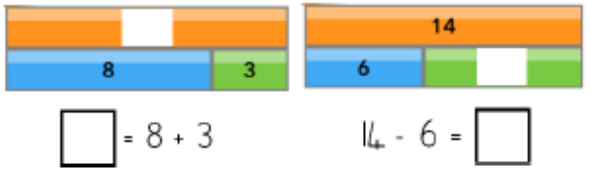
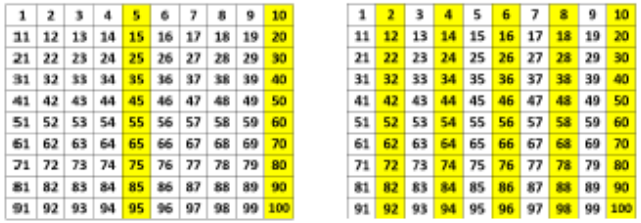
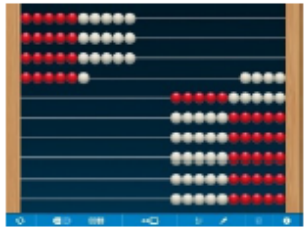
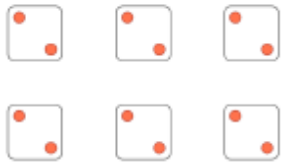
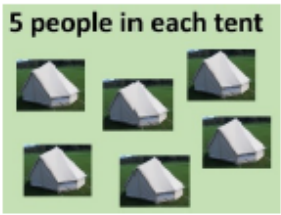
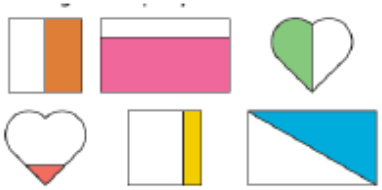
Reception

Objective	Visual representations
Count reliably with numbers from 1-20	For 1:1 counting, number sounds are clearly separated and items counted with exaggerated movements. Counted objects are rearranged in regular patterns to support quantity recognition.  Children learn that each object is counted once and the last number is the total for the set— count small sets in irregular arrangements. Progress by counting out items from larger set; objects that can't be moved; make objects not visible once counted; count movements and sounds. Counting on taught by counting two sets, then screening one of the counted sets.   
Identify and use numerals	Children match numerals to different representations of number for quantities 1-10 (see 'knowledge of numbers as quantities') e.g. making and finding 5 in different ways. Children learn that 'teen' represents 10 and match teen/ten visual cards. Place value arrow cards used for partitioning and combining tens and units.  *Different representations matched to numerals*  
Understand 10 as a unit	Items are counted into groups of 10, for example pipe cleaners bundled into 10s or items counted into 10-frames. Children recognise quantities in multiple 10-frames as 'how many tens, how many ones'.   Children count tens/ones on Slavonic Abacus. Coloured 100-square supports counting in tens.  

Objective	Visual representations
Secure knowledge of numbers as quantities	Children instantly subitize 1-3 items through dot pattern games and everyday experiences. Items may be unrelated.  *Image shown briefly. How many toys?* A range of representations used for quantities 1-10. Children show numbers in different ways on fingers; games used to improve finger discrimination. Quick recognition of regular and irregular dot patterns, with larger quantities visualised in two parts (e.g. see 5 as 3 and 2). Children are taught to recognise quantities on 10-frame and base-5 number track. 
To recite forwards and backwards number word sequences	Forwards and backwards number word sequences supported using songs and rhymes. Children continue number sequences starting from different numbers with some prior words in appropriate range e.g. 3, 4, 5, 6... or 24, 23... The transition ofer 10s boundaries supported by visuals. Number tracks used, with numbers hidden to add challenge as appropriate. 
Add and subtract single-digit numbers	Addition built on experience of counting two groups. Opportunities provided for comparing quantities, using language more/less. Combining quantities in 10-frames and using Numicon encourage non-counting-in-ones strategies. Arrangement of sets counted also encourage counting on and calculation strategies.  *Representation of 4+3 encourages counting on from 4* *Representation of 4+3 to help visualise 3+3+1*
Develop pre-multiplication and division concepts	Halves and doubles identified in a range of contexts, with a focus on equal halves. Shown on 10-frames and with Numicon.  Counting in 2s supported by colouring of 100-square  Opportunities for 'repeat add' counting in context e.g. counting socks. Repeated addition shown with dice patterns. Grouping and sharing context tasks provided.  

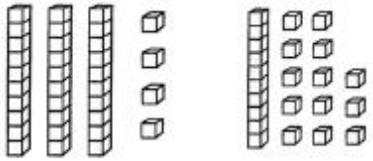
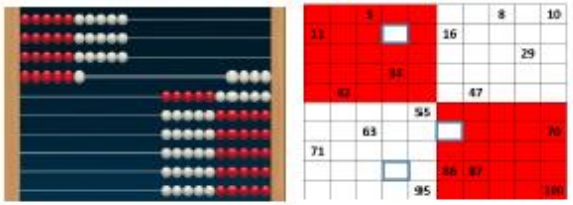
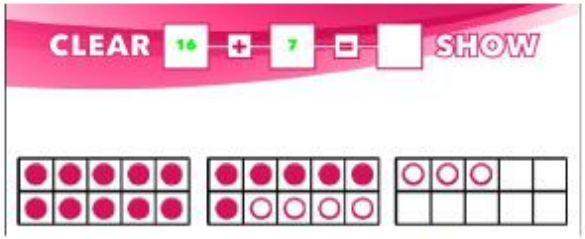
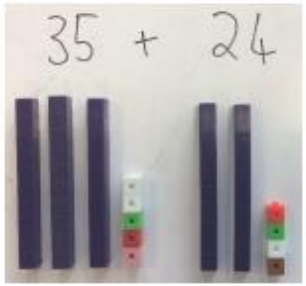
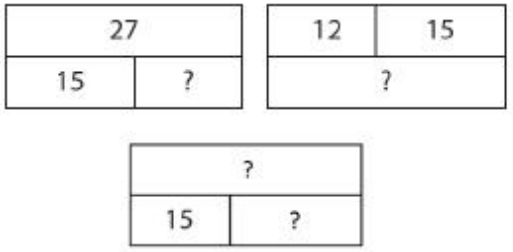
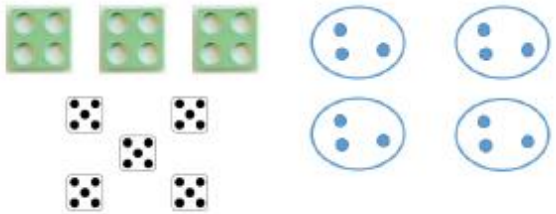
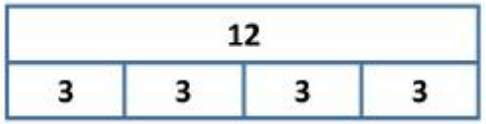
Year 1

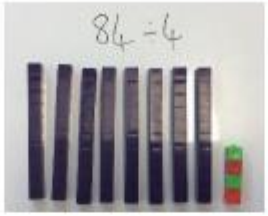
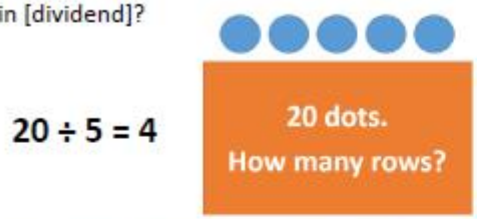
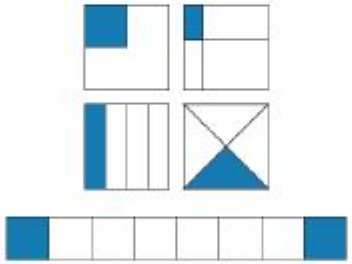
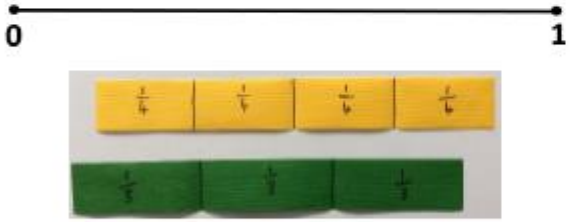
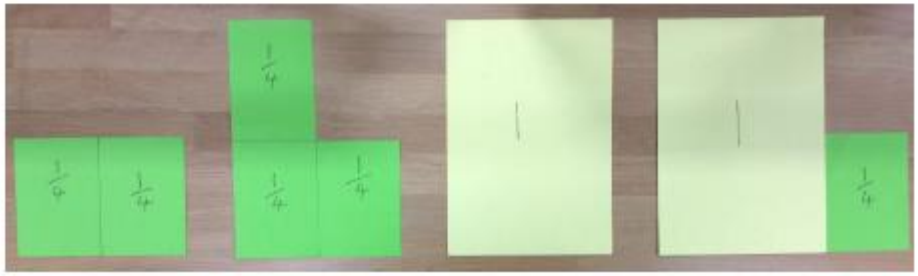
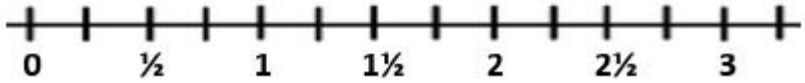
Objective	Visual representations
Know 1 more/less in the range 1-100, focusing on bordering tens boundaries	Identify and show one more/less in different ways. Example game: one more/less bingo.  Find missing numbers on number track, focusing on tens boundaries.  Slavonic Abacus to show quantities 1-100 (iPad app 'Number Rack'). 
With visuals, discern teens from tens	Organise large quantities in groups of 10 e.g. with egg boxes or pipe cleaners.  Use teens/tens matching cards.  Identify and make 2-digit numbers with dienes, showing in different ways.  Partition 2-digits numbers using place-value cards 
Able to represent 1-10 in a range of ways, working out small quantities without counting all items	Immediate recognition of Numicon, 10-frame images, tally charts, dot patterns and finger patterns.  Represent numbers on fingers in different ways.  Estimate position of numbers on blank number lines with different start/end numbers. 
Break down 1-10 in all possible ways, write number sentences using +, - and =	Subitizing games for regular and irregular dot patterns, with children visualising quantities in two parts.  Arrangement of 2 colours of items e.g. in egg box 10-frame or with Numicon.  Introduction of part-whole model from individuals squares/items to bars. 

Objective	Visual representations
Represent and use number bonds and related subtraction facts within 20	10-frames and 2-colour number tracks show calculations bordering 10: 'how many to 10, how many more?' Lead to use of blank number line.  Equivalence shown with balance scales and dice patterns.  Bar models used to show relationship between addition and subtraction. 
Count in multiples of 2, 5 and 10	100-square with columns highlighted used to support counting. The Slavonic Abacus (iPad app 'Number Rack') used to visualise quantity when counting.   Count in visual then hidden groups of 2, 5 and 10.  
Recognise and make one-half in a range of ways (discern examples from non-examples); identify one-quarter	Half of a shape/capacity , number of objects, 10-frame half/double, half of length, half of an amount of money.  Colour half of each whole shape:  Circle half of this group of strawberries.  What is half of this amount?  
Link the value of coins to a matching visual	Match value of coins to Numicon pieces, use Numicon to support calculations involving money. 

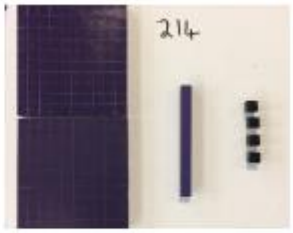
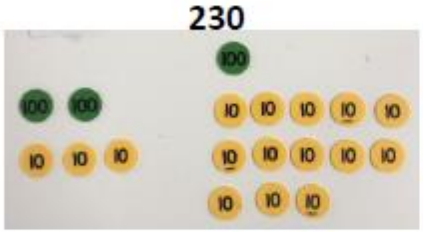
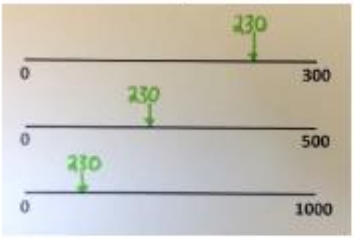
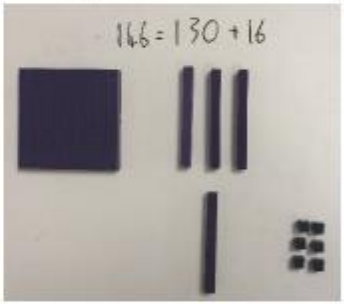
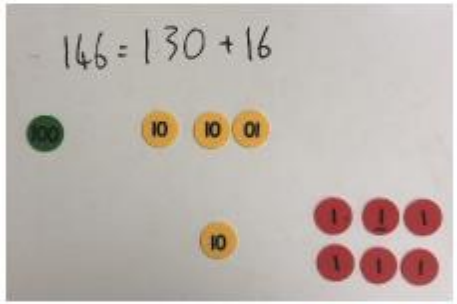
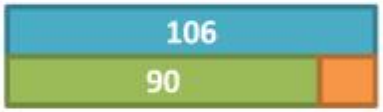
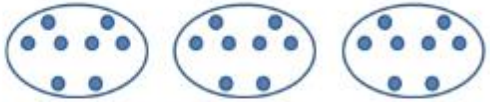
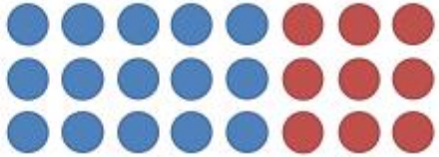
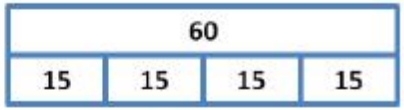
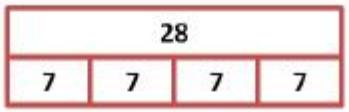
VISUAL REPRESENTATIONS

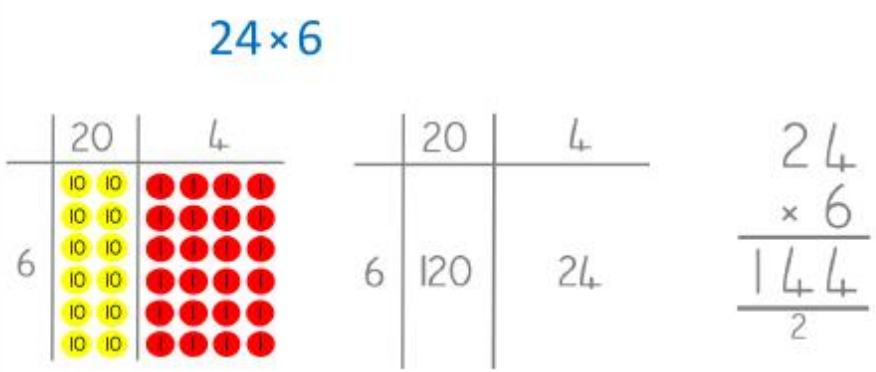
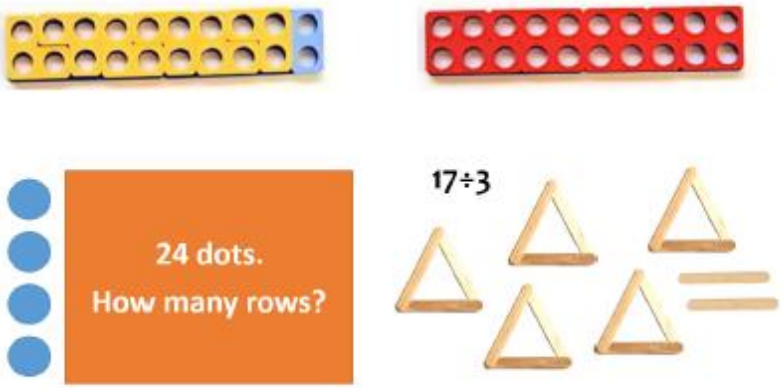
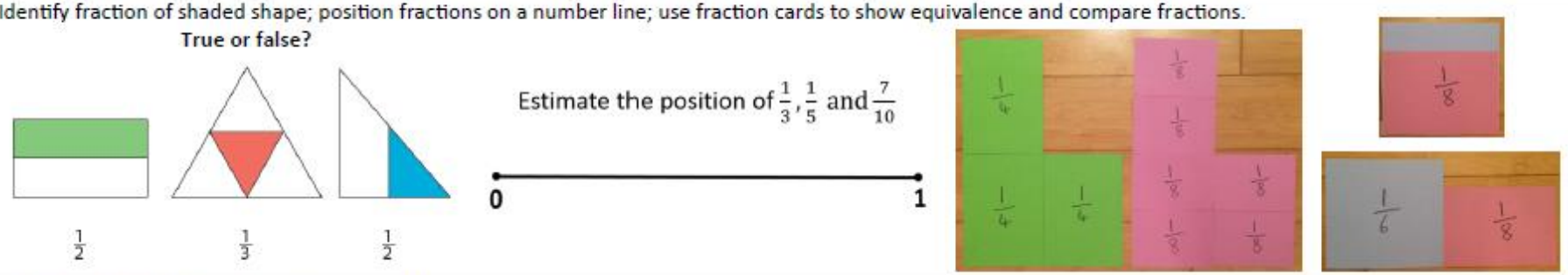
Year 2

Objective	Visual representations
Represent numbers 1-100 in a range of ways, showing understanding of place value	Represent tens/teens using dienes, showing numbers in different ways.  Partition 2-digits numbers using place-value cards.  Estimate position of numbers on blank number lines with different start/end.  Recognise amount on Slavonic Abacus, seeing tens and ones; find missing numbers on 100-square. 
Use different calculation strategies for adding and subtracting one and two-digit numbers	Calculation within 30 using 10-frames, lead to use of number line, e.g. use egg-box 10-frames and app 'I See Addition and Subtraction'.   Model calculation using partitioning with dienes.  Bar modelling to show relationship between + and – (using words 'whole/parts'). Include spatial reasoning estimates. 
Understand x as repeated adding, find related x and ÷ facts from a number sentence	Numicon and images of repeated quantities show multiplication as repeated addition.  Arrays show commutativity of multiplication. Columns/rows circled to link to division.  Bar model shows relationship between whole/ parts and makes links to division. 

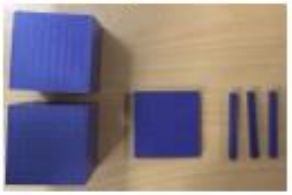
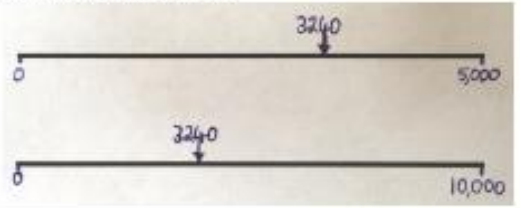
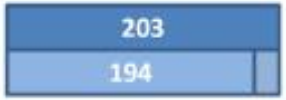
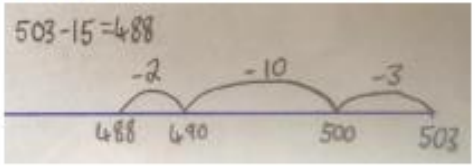
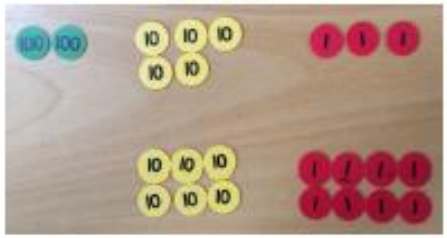
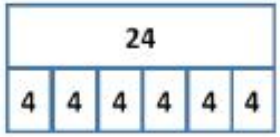
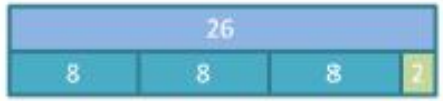
Objective	Visual representations		
Use sharing and grouping strategies for division, re-late division to finding unit fractions of quantities	Sharing supported by appropriate visuals, used where a large total is shared into few groups:  	Grouping strategy modelled with covered arrays and Numicon: how many [divisors] in [dividend]? **20 ÷ 5 = 4** **20 dots.** **How many rows?**  	Grouping context questions with supporting visuals. **How many cars are needed to take 18 children to the match? 4 children per car.** 
Represent fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ in a range of ways; order and recognise equivalence.	Fractions of areas/objects (and non-examples): **Which of these diagrams are ¼ blue?**  *Include fractions of containers*	Fractions of a length/number line: Estimate the position of $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{3}{4}$  	Fractions of quantity: **The children can have ¾ of the cupcakes.** 
Use halves and quarters as counting numbers, going over 1	Modelled with fraction cards and on a number line. 	 	

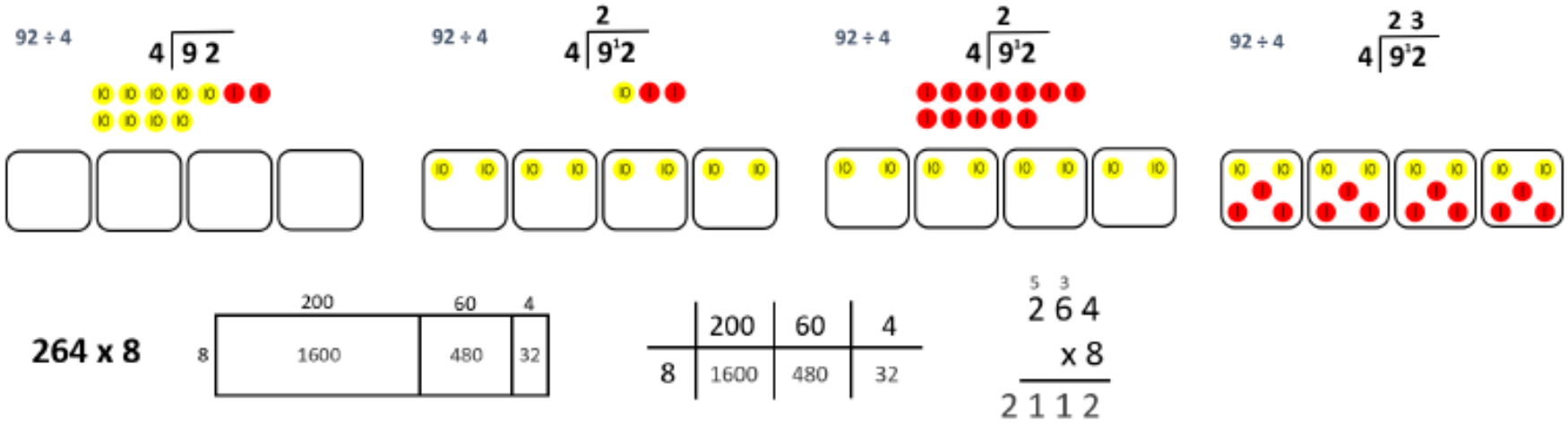
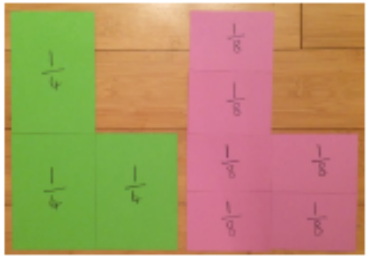
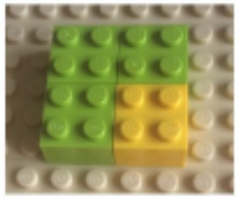
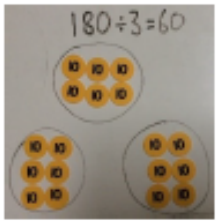
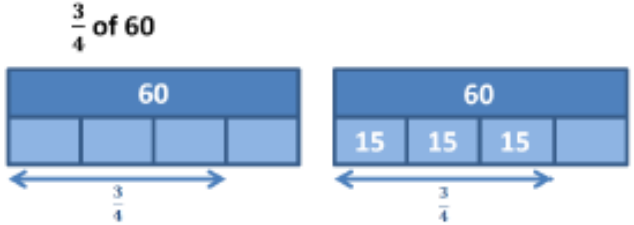
Year 3

Objective	Visual representations
Represent 3-digit numbers in a range of ways, showing an understanding of place value	Make 3-digit numbers using dienes and place value cards, showing how they can be partitioned.   Make the same number in different ways with place value coins.  Estimate position of numbers on blank number lines with different start/end numbers. 
Add and subtract ones, tens and hundreds to HTU, making realistic estimates	Dienes, place value coins and app 'I See Addition and Subtraction' model written addition and subtraction. Bar model shows subtraction as difference.    106 – 90 = 
Understand the inverse relationship between x and ÷; know x as repeated adding, use to derive related multiplication facts.	A range of images show multiplication as repeated addition. 2-colour arrays show distributive law. 8x3    5x3 + 3x3 Bar model shows link between multiplication and division, and model division as sharing and grouping. 60 ÷ 4 = 15  *'60 in four equal parts'* 28 ÷ 7 = 4  *'How many 7s in 28?'*

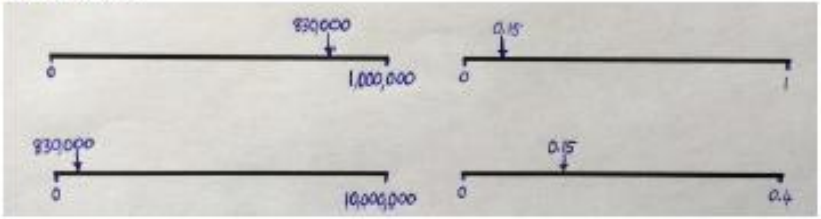
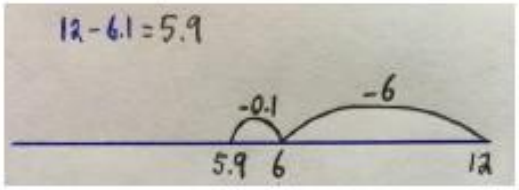
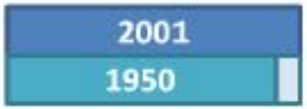
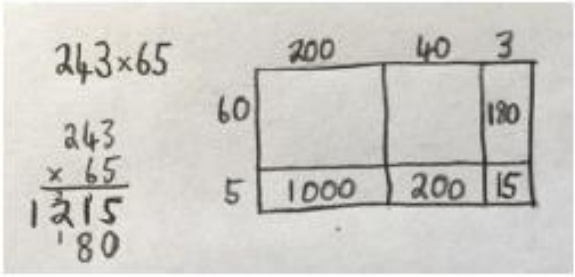
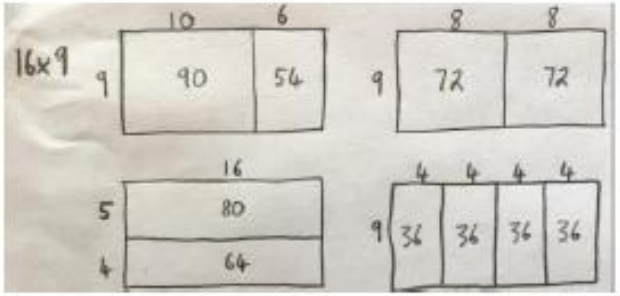
Objective	Visual representations
Use efficient formal written methods for multiplication and division	Multiplication modelled using place value coins, leading to efficient written forms:  The concept of 'How many [divisors] in [dividend]' shown using Numicon, part-hidden arrays and by making shapes with matchsticks. **20 ÷ 3 (how many 3s in 20?) and 20 ÷ 5 (how many 5s in 20?):** 
Simple unit/non-unit fractions represented in a range of ways; different fractions compared including equivalence	Identify fraction of shaded shape; position fractions on a number line; use fraction cards to show equivalence and compare fractions. 
Use quarters, halves and tenths as counting numbers going over 1	Modelled with fraction cards and on number lines. 

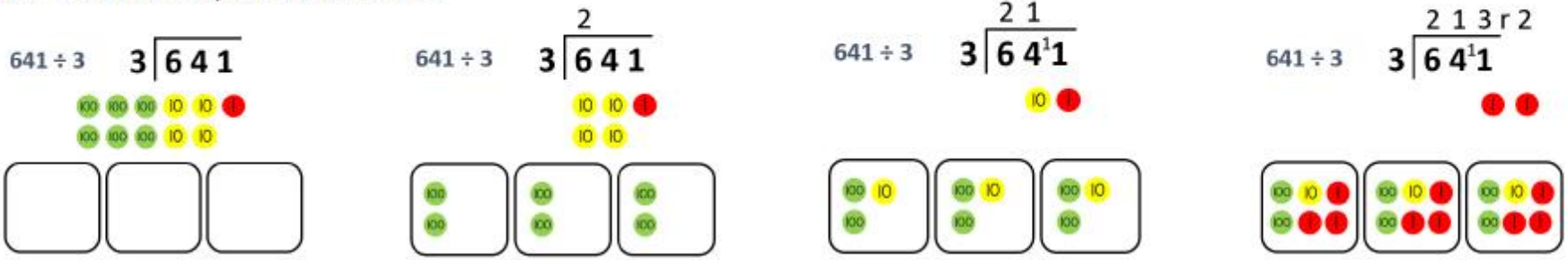
Year 4

Objective	Visual representations
Represent 4-digit numbers in a range of ways, showing understanding of place value	Make 4-digit numbers using dienes and place value coins, building numbers in different ways. *2,130*  *420 with three 100s and twelve 10s*  Estimate the position of numbers on blank number lines with different start/end numbers. 
Choose efficient mental strategies for adding and subtracting numbers	Round and adjust to calculate, model with appropriate visual *350-198 modelled with place value counters: take away 200, add 2.*   Choose whether to count on or count back, show with number line or bar model. **203 - 194**  
Become fluent in written methods for addition and subtraction	Model vertical methods for addition and subtraction step-by-step using place value counters and iPad app 'I See Addition and Subtraction'. 253 + 68  
Understand and represent multiplication and division in a range of ways; derive related facts from a given calculation.	Use arrays and bar models to derive related multiplication and division facts This image shows **4 x 6**  *Change the image to show **4 x 7*** This image shows **4 x 6**  *Use the image to calculate **4 x 12*** Understand division as 'how many [divisors] in [dividend]' showing remainders using matchsticks to make shapes and bar models. **17÷3**  

Objective	Visual Representations
Use efficient formal written methods for multiplication and division of 3-digit numbers	Division modelled with place value counters. Written multiplication represented by area model—links made between grid method and compact method. 
Find equivalent fractions, calculate fractions of amounts (unit and non-unit fractions)	Fraction cards and Lego used to show equivalence.   $\frac{3}{4} = \frac{12}{16}$ Fractions of quantities shown using place value counters and bar models, presented in stages.  
Know decimal equivalents for quarters and halves, relating to division	Dividing length of a metre ruler into two/four equal parts. 

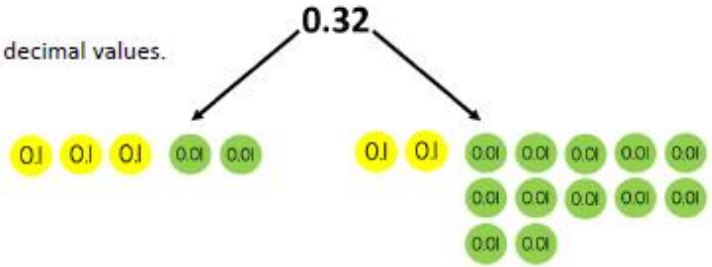
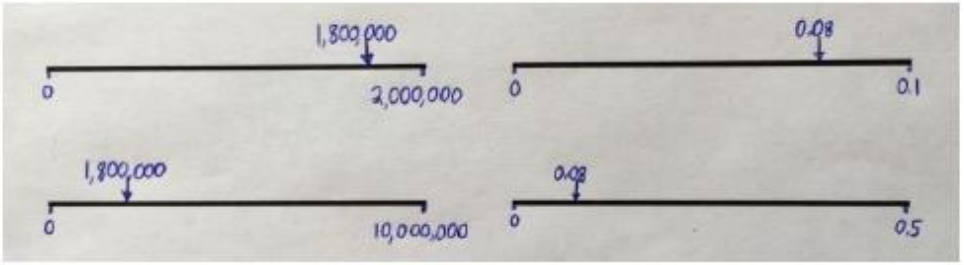
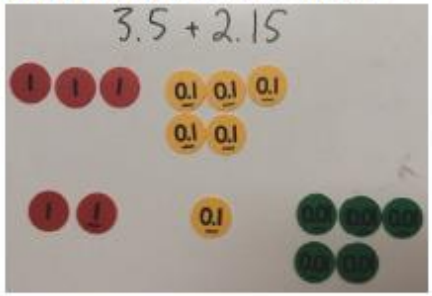
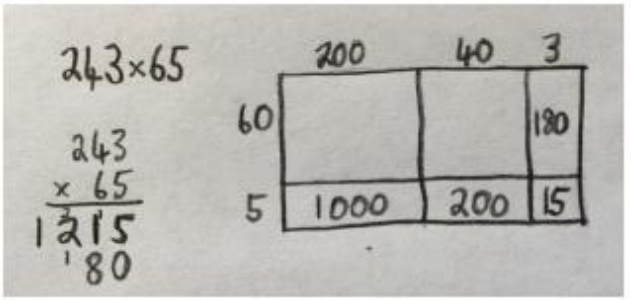
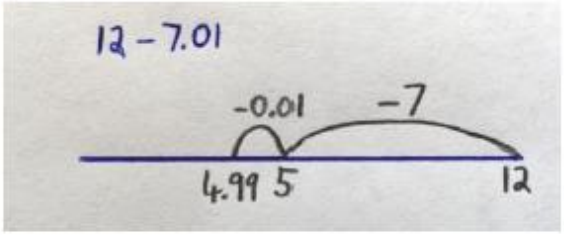
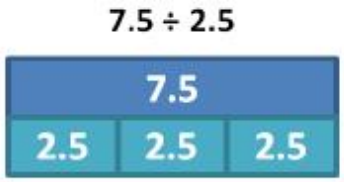
Year 5

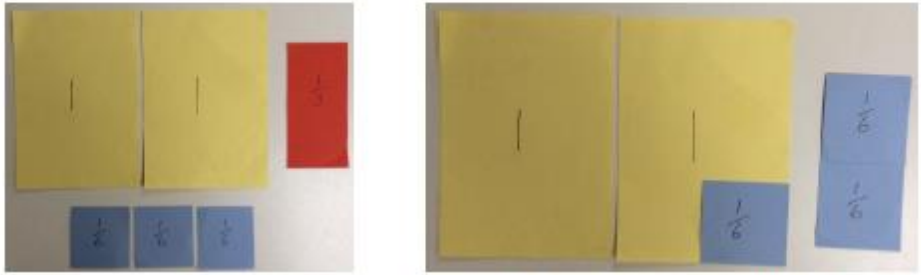
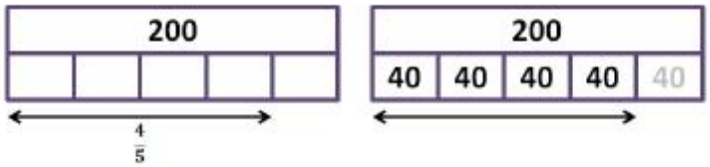
Objective	Visual representations	
Represent the value of digits in numbers of up to 7-digits and decimals to thousandths	Make numbers in the range using place value coins, partitioning decimal values and showing the same number in different ways. *0.35*  *430* 	Estimate the position of numbers on blank number lines with different start/ end numbers. 
Choose efficient strategies and apply knowledge of place value when adding and subtracting	Model vertical methods for addition and subtraction step-by-step using iPad app 'I See Addition and Subtraction' or place value counters.  	Mental calculation methods modelled using appropriate visual, e.g. rounding and adjusting on a number line, bar model to show subtraction as difference.  **2001 - 1950** 
Develop a range of strategies for multiplication including efficient written methods	Compact written method made visual by area model. 	Area model used to show multiplication where numbers are partitioned in different ways. 

Objective	Visual representations
Develop a range of strategies for division including efficient written methods	Division modelled with place value counters.  Bar model used to reinforce 'how many [divisors] in [dividend]?' **750 ÷ 150** 750 150 150 150 150 150
Compare and order fractions, find equivalent fractions, add and subtract fractions.	Fraction cards used to compare, show equivalence and model calculations.   *Example: ¾ + ½* 
Find decimal equivalents for quarters, fifths and tenths, relating to division	Dividing length of a metre ruler into two/four/five equal parts. 

VISUAL REPRESENTATIONS

Year 6

Objective	Visual representations
Represent numbers of up to 8-digits and decimals to thousandths in a range of ways	Make numbers in a range of ways using place value coins, partitioning decimal values.  Estimate the position of numbers on blank number lines with different start/end numbers. 
Carry out formal written methods of calculation for all four operations	Place value of numbers in addition and subtraction modelled using place value counters.  Multiplication visualised using area model. 
Choose efficient strategies and apply flexible knowledge of number to calculate	Choose appropriate visuals to model structure of calculations, including modelling worded questions.   

Objective (Y6)	Visual representations
Add and subtract fractions with different denominators	Fraction cards to show conversion into common denominators and calculating over whole-number boundaries. Example: $2\frac{1}{3} - \frac{3}{6}$ 
Multiply and divide unit fractions and simple non-unit fractions	Area model diagrams to model a fraction being divided or multiplied by a fraction (modelled in two steps). 
Calculate percentages and fractions of quantities	Bar model visualises finding fraction/percentage of quantity and finding the whole given a percentage/fraction. Shown step-by-step. $\frac{4}{5}$ x 200  40% of a number is 60. What's the number? 
Describe linear number sequences, including using formulae in the form y = mx + c	Numicon and bar model used to model linear number sequences or equations.  y = 3x + 5